Sirius Mathematical Centre

International Conference

Spectral Theory and Mathematical Physics

3–7 February 2020

Programme and Abstracts

Sochi 2020

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Speakers:

- Alexander Aptekarev, *Keldysh Institute of Applied Mathematics RAS, Russia*
- Matteo Capoferri, *University College London, United Kingdom*
- Mark Dorodnyi, *St. Petersburg State University, Russia*
- Alexandre Fedotov, *St. Petersburg State University, Russia*
- Tatiana Garmanova, *Lomonosov Moscow State University, Russia*
- Amru Hussein, *Technische Universität Kaiserslautern, Germany*
- Aleksei Ilyin, *Keldysh Institute of Applied Mathematics RAS, Moscow, Russia*
- Frédéric Klopp, *IMJ-PRG, Sorbonne Université, France*
- Yuri Kordyukov, *Ufa Federal Research Centre, Russia*
- Vadim Kostrykin, *Johannes Gutenberg-Universität Mainz, Germany*
- Michael Levitin, *University of Reading, United Kingdom*
- Vladimir Lysov, *Keldysh Institute of Applied Mathematics RAS, Russia*
- Yoshihisu Miyanishi, *Osaka University, Japan*
- Alexander Nazarov, *PDMI RAS; St. Petersburg State University, Russia*
- Vadim Ognov, *IMJ-PRG, Sorbonne Université, France*
- Leonid Pastur, *Institute for Low Temperature Physics and Engineering, Kharkov, Ukraine*
- Svetlana Pastukhova, *Russian Technological University, Russia*
- Alexander Poretskii, *St. Petersburg State University, Russia*
- Nikita Rastegaev, *St. Petersburg State University, Russia*
- Grigori Rozenblum, *St. Petersburg State University, Russia*
- Albrecht Seelmann, *Technische Universität Dortmund, Germany*
- Nikita Senik, *St. Petersburg State University, Russia*
- Ekaterina Shchetka, *St. Petersburg State University, Russia*
- Igor Sheipak, *Lomonosov Moscow State University, Russia*
- Andrei Shkalikov, *Lomonosov Moscow State University, Russia*
- Vladimir Sivkin, *Lomonosov Moscow State University, Russia*
- Vladimir Sloushch, *St. Petersburg State University, Russia*

- Dimitri Yafaev, *St. Petersburg State University, Russia*
- Sylvain Zalczer, *Université de Toulon, La Garde, France*
- Ekaterina Zlobina, *St. Petersburg State University, Russia*

CONFERENCE PROGRAMME

MONDAY 3 February:

9:00–9:30: REGISTRATION

9:30–10:20: Leonid Pastur (B. I. Verkin Institute for Low Temperature Physics and Engineering, Kharkov, Ukraine). *Analogs of Szegö's theorem for ergodic operators.*

10:30–11:20: Frédéric Klopp (Sorbonne Université, France). *Exponential decay for the 2 particle density matrix of disordered many-body fermions at zero and positive temperature.*

COFFEE BREAK

11:50–12:20: Vadim Ognov (IMJ-PRG, Sorbonne Université, France). *Luttinger-Sy model and ground state energy per particle in the thermodynamic limit.*

12:20–12:50: Albrecht Seelmann (Technische Universität Dortmund, Germany). *Anderson localization beyond regular Floquet eigenvalues.*

LUNCH

14:30–15:00: Sylvain Zalczer (Université de Toulon, La Garde, France). *Anderson localization for random Dirac operators.*

15:00–15:30: Matteo Capoferri (University College London, United Kingdom). *Global hyperbolic propagators in curved space.*

COFFEE BREAK

16:00–16:30: Alexander Poretskii (St. Petersburg State University, Russia). *Mathematical scattering theory in quantum waveguides.*

16:30–17:00: Nikita Rastegaev (St. Petersburg State University, Russia). *On spectral asymptotics for the Sturm–Liouville problem with singular self-similar weight measure.*

17:30 WELCOME PARTY

TUESDAY 4 February:

9:30–10:20: Dimitri Yafaev (St. Petersburg State University, Russia). *Asymptotic behavior of orthogonal polynomials without the Carleman condition*

10:30–11:20: Grigori Rozenblum (St. Petersburg State University, Russia). *Spectral properties of the Neumann–Poincaré operator for the elasticity system and related questions about zero order pseudodifferential operators.*

COFFEE BREAK

11:50–12:40: Yoshihisu Miyanishi (Center for Mathematical Modeling and Data Science, Osaka University, Japan). *Applications of Neumann–Poincaré operators: non-cloaking by anomalous localized resonance for the electro-static system in three-dimensional smooth convex domains.*

LUNCH

14:30–15:20: Svetlana Pastukhova (Russian Technological University, Russia). *Modified method of the first approximation and operator-type estimates in homogenization.*

COFFEE BREAK

15:50–16:20: Vladimir Sloushch (St. Petersburg State University, Russia). *Estimates and asymptotic behavior of the discrete spectrum of a discrete periodic Schrodinger operator perturbed by a decreasing potential.*

16:20–16:50: Mark Dorodnyi (St. Petersburg State University, Russia). *Homogenization of hyperbolic equations with periodic coefficients.*

WEDNESDAY 5 February:

9:00–9:50: Michael Levitin (University of Reading, United Kingdom). *Asymptotics of Steklov eigenvalues in curvilinear polygons.*

10:00–10:50: Alexander Nazarov (St. Petersburg Dept. of Steklov Mathematical Institute and St. Petersburg State University, Russia). *Spectral asymptotics for some problems generated by the FBM-like processes.*

COFFEE BREAK

11:20–12:10: Aleksei Ilyin (Keldysh Institute of Applied Mathematics RAS, Russia). *Lieb–Thirring and Ladyzhenskaya inequalities on the sphere and on the torus.*

LUNCH

13:20 TRIP TO KRASNAYA POLYANA

THURSDAY 6 February:

9:30–10:20: Alexander Aptekarev (Keldysh Institute of Applied Mathematics RAS, Russia). *On spectrum of a selfadjoint difference operator on a graph-tree.*

10:30–11:20: Andrei Shkalikov (Lomonosov Moscow State University, Russia). *On perturbations of self-adjoint and normal operators. Analytical aspects.*

COFFEE BREAK

11:50–12:40: Igor Sheipak (Lomonosov Moscow State University, Russia). *Constants in the Sobolev embedding theorems. Applications to spectral problems*

LUNCH

14:30–15:00: Tatiana Garmanova (Lomonosov Moscow State University, Russia). *Properties of estimation functions in inequalities of Friedrichs–Markov–Kolmogorov type.*

15:00–15:30: Vladimir Sivkin (Lomonosov Moscow State University, Russia). *Preservation of the basis property under locally subordinated perturbations of self-adjoint operators.*

COFFEE BREAK

16:00–16:30: Vladimir Lysov (Keldysh Institute of Applied Mathematics RAS, Russia). *Direct and inverse problems for vector logarithmic potentials with external fields.*

16:30–17:00: Amru Hussein (Technische Universität Kaiserslautern, Germany). *Non-self-adjoint graphs: spectra, similarity, semigroups.*

FRIDAY 7 February:

9:30–10:20: Vadim Kostrykin (Johannes Gutenberg-Universität Mainz, Germany). *On the invertibility of block matrix operators.*

10:30–11:20: Yuri Kordyukov (Institute of Mathematics with Computing Centre, Ufa Federal Research Centre, RAS, Russia). *Semiclassical eigenvalue asymptotics for the magnetic Laplacian with full-rank magnetic field.*

COFFEE BREAK

11:50–12:40: Alexandre Fedotov (St. Petersburg State University, Russia). *The spectrum and density of states of the almost Mathieu operator in semiclassical approximation.*

LUNCH

14:30–15:00: Ekaterina Shchetka (St. Petersburg State University, Russia). *On semiclassical methods for difference equations.*

15:00–15:30: Ekaterina Zlobina (St. Petersburg State University, Russia). *High-frequency diffraction by a non-smooth contour.*

COFFEE BREAK

Abstracts of talks

On the spectrum of a selfadjoint difference operator on a graph-tree

Alexander Aptekarev

Keldysh Institute of Applied Mathematics, Russia

We consider a class of selfadjoint discrete Schrödinger operators defined on an infinite homogeneous rooted graph-tree. The potential of this operator consists of the coefficients of the Nearest Neighbor Recurrence Relations for the Multiple Orthogonal Polynomials (MOPs).

For the general class of potentials generated by Angelesco MOPs we prove that the essential spectrum of these operators is a union of the supports of the components of the vector orthogonality measure $\vec{\mu} := (\mu_1, \ldots, \mu_d)$ for the Angelesco MOPs.

This is a joint work with Sergey Denisov (Madison University) and Maxim Yattselev (IUPUI).

Global hyperbolic propagators in curved space

Matteo Capoferri

University College London, United Kingdom

Consider a hyperbolic linear partial differential equation (PDE) or system of PDEs. The propagator is the linear operator mapping initial conditions (Cauchy data) to the solution of the hyperbolic equation or system. Our aim is to construct explicitly, modulo smooth terms, propagators for physically meaningful PDEs and systems of PDEs on manifolds without boundary, and to do this in a global (i.e. as a single oscillatory integral) and invariant (under changes of local coordinates and any gauge transformations that may be present) fashion. Here by "explicitly" we mean reducing the PDE problem to integration of ordinary differential equations. The crucial element in our global construction is the use of a complex-valued, as opposed to real-valued, phase function — an idea proposed by Laptev, Safarov and Vassiliev in the nineties — which allows one to circumvent topological obstructions due to caustics. The three mathematical models discussed in the talk are the wave equation, the massless Dirac equation and Maxwell's equations.

The talk is partly based on joint work with D. Vassiliev (UCL) and M. Levitin (Reading).

Homogenization of hyperbolic equations with periodic coefficients

Mark Dorodnyi

St. Petersburg State University, Russia

In $L_2(\mathbb{R}^d; \mathbb{C}^n)$, we consider a self-adjoint matrix strongly elliptic operator A_ε, $\varepsilon > 0$ given by the differential expression $b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$. Here $g(\mathbf{x})$ is a periodic bounded and positive definite matrix-valued function, and $b(\mathbf{D})$ is a first order differential operator. We study the behavior of the operators $\cos(tA_\varepsilon^{1/2})$ and $A_\varepsilon^{-1/2} \sin(tA_\varepsilon^{1/2})$, $t \in \mathbb{R}$, for small ε. It is proved that, as

$\varepsilon \to 0$, these operators converge to $\cos(t(A^0)^{1/2})$ and $(A^0)^{-1/2}\sin(t(A^0)^{1/2})$, respectively, in the norm of operators acting from the Sobolev space $H^s(\mathbb{R}^d;\mathbb{C}^n)$ (with a suitable s) to $L_2(\mathbb{R}^d;\mathbb{C}^n)$. Here $A^0 = b(\mathbf{D})^* g^0 b(\mathbf{D})$ is the effective operator. We prove sharp-order error estimates and study the question about the sharpness of the results with respect to the norm type as well as to the dependence on t. The results are applied to study the behavior of the solution $\mathbf{u}_\varepsilon(\mathbf{x},t)$ of the Cauchy problem for the hyperbolic equation $\partial_t^2 \mathbf{u}_\varepsilon(\mathbf{x},t) = -(A_\varepsilon \mathbf{u}_\varepsilon)(\mathbf{x},t)$. The talk is based on a joint work with Tatiana Suslina.

The spectrum and density of states of the almost Mathieu operator in semiclassical approximation

Alexandre Fedotov

St. Petersburg State University, Russia

The almost Mathieu operator with a frequency represented by an infinite continued fraction with sufficiently large elements is discussed. The spectrum of this operator is a Cantor set and can be described by sequentially “removing” finite sets of shorter and shorter gaps. Intervals between the gaps found at each step are called bands. Asymptotics of the centers and lengths of most of the bands are described, the values of the integrated density of states are calculated in the adjacent gaps, asymptotic properties of distribution of its values over gaps are described. The results were obtained by semiclassical methods.

The talk is based on a joint work with Ekaterina Shchetka (St. Petersburg State University and St. Petersburg Dept. of Steklov Institute).

Properties of estimation functions from inequalities of Friedrichs–Markov–Kolmogorov type

Tatiana Garmanova

Lomonosov Moscow State University, Russia

We study the functions $A_{n,k}$ that lead to the best possible estimates of the form

$$|f^{(k)}(x)| \leqslant A_{n,k}(x)\|f^{(n)}\|_{L_2[0;1]}$$

with any $f \in \mathring{W}_2^n[0;1]$. Recurrence formulas for $A_{n,k}^2$ were obtained, and a relation for $A_{n,k}^2$ with the primitives of Legendre polynomials was found. We also investigate properties of the estimation functions’ maxima, the location of which is closely related to the problem of finding the sharp constants in the Sobolev embedding theorems. An explicit representation of $A_{n,k}^2$ is also given in terms of the hypergeometric functions.

Non-self-adjoint graphs: spectra, similarity, semigroups

Amru Hussein

Technische Universität Kaiserslautern, Germany

On finite metric graphs, Laplace operators subject to general non-self-adjoint matching conditions imposed at graph vertices are considered. A regularity criterion is proposed and spectral properties of such regular operators are investigated, in particular similarity transforms to self-adjoint operators and generation of C_0-semigroups. Concrete examples are discussed exhibiting that non-self-adjoint boundary conditions can yield to unexpected spectral features.

The talk is based on joint works with David Krejčiřík (Czech Technical University in Prague), Petr Siegl (Queen's University Belfast) and Delio Mugnolo (FernUniversität Hagen).

The Lieb–Thirring and Ladyzhenskaya inequalities on the sphere and on the torus

Alexei Ilyin

Keldysh Institute of Applied Mathematics, Russia

We prove the Lieb–Thirring inequalities in the dual formulation for orthonormal families of scalar and vector functions on the 2d sphere and on the 2d torus (with equal periods). We obtain a rather good estimate of the constant. In both cases, we use the discrete version of the modified Rumin's method.

This is a joint work with A. A. Laptev and S. V. Zelik.

Exponential decay for the 2 particle density matrix of disordered many-body fermions at zero and positive temperature

Frédéric Klopp

Sorbonne Université, France

We will consider a simple model for interacting fermions in a random background at zero and positive temperature. At zero temperature, we prove exponential decay for the 2 particle density matrix of a ground state. At positive temperature, we prove exponential decay for the 2 particle density matrix of the density operator in the grand canonical ensemble.

Semiclassical eigenvalue asymptotics for the magnetic Laplacian with a full-rank magnetic field

Yuri Kordyukov

Ufa Federal Research Centre, Russia

We consider the semiclassical magnetic Laplacian with a full-rank magnetic field on a closed manifold (or, equivalently, the Bochner Laplacian on high tensor powers of a positive line bundle on a closed symplectic manifold). We assume that the magnetic field has discrete wells and prove asymptotic expansions for low-lying eigenvalues of the operator. The proof uses the algebra of Toeplitz operators associated with the Bochner Laplacian. This algebra provides a Berezin–Toeplitz type quantization of the symplectic manifold. We will describe its construction and discuss some spectral properties of self-adjoint Toeplitz operators.

On the invertibility of block matrix operators

Vadim Kostrykin

Johannes Gutenberg-Universität Mainz, Germany

We consider self-adjoint block matrix operators of the form

$$B = \begin{pmatrix} A_0 & V \\ V^* & A_1 \end{pmatrix}$$

acting on the orthogonal sum of two Hilbert spaces $\mathfrak{H}_0 \oplus \mathfrak{H}_1$. We treat the following cases: (i) the diagonal part $A_0 \oplus A_1$ of the operator B is invertible, (ii) the diagonal part $A_0 \oplus A_1$ is not invertible but A_0 and A_1 are subordinated in sense that $A_1 \leq 0 \leq A_0$. In both cases we prove new criteria for the invertibility of the operator B and obtain a lower bound on the width of the gap containing the point zero.

The talk is based on a joint work with Lukas Rudolph.

Asymptotics of Steklov eigenvalues for curvilinear polygons

Michael Levitin

University of Reading, United Kingdom

I will discuss sharp asymptotics of large Steklov eigenvalues for planar curvilinear polygons. The asymptotic expressions for eigenvalues are given in terms of roots of some trigonometric polynomials which depend explicitly on the side lengths and angles of the polygon.

It turns out that both the eigenvalue asymptotics and the corresponding quasimodes depend non-trivially on the arithmetic properties of the angles of the polygon, and are also related to the eigenvalues of a particular quantum graph. The proofs involve some classical hydrodynamics results related to a sloping beach problem, and a sloshing problem. I'll also state some open questions. The talk will be based on joint works with Leonid Parnovski, Iosif Polterovich, and David Sher, see arXiv:1908.06455 and arXiv:1709.01891.

Direct and inverse problems for vector logarithmic potentials with external fields

Vladimir Lysov

Keldysh Institute of Applied Mathematics, Russia

We discuss some extremal problems for the energy of the logarithmic potential with external fields. The problems are closely related to the inverse spectral problem method. The method is based on the relations between the external field and the supports of the equilibrium measures which were discovered in two pioneering papers by Buyarov–Rakhmanov and Mhaskar–Saff. We propose a generalization of this method for the vector of measures with matrix of interaction between components. The talk is based on a joint work with A. I. Aptekarev and M. A. Lapik.

Applications of Neumann-Poincaré operators: non-cloaking by anomalous localized resonance for the electro-static system in three dimensional smooth convex domains

Yoshihisu Miyanishi

Center for Mathematical Modeling and Data Science, Osaka University, Japan

The Neumann-Poincaré (NP) operator is an integral operator defined on the boundary of a bounded domain. It arises naturally when solving the Dirichlet and Neumann boundary value problems for the Laplacian in terms of layer potentials. As the name suggests, its study goes back to Neumann and Poincaré. It was a central object in the Fredholm theory of integral equations and the theory of singular integral operators. There is rapidly growing interest in the spectral theory of the NP operator in relation to plasmonics. The spectral theory of the NP operator has also been applied to analysis of cloaking by anomalous localized resonance on the plasmonic structure. The purpose of this talk is to introduce the notion of plasmonics and the spectral theory of the NP operator, and compare them in a quantitatively precise way. Furthermore we are interested in the anomalous localized resonance (ALR) and prove that it doesn't occur in strictly convex smooth three-dimensional bounded domains. As a consequence, resonance occurs only at corresponding eigenvalues of NP operator on such domains. These results are proved using the pseudo-differential operators' calculus and the spectrum of NP operator.

Spectral asymptotics for some problems generated by the FBM-like processes

Alexander Nazarov

St. Petersburg Dept. of Steklov Math. Institute and St. Petersburg State University, Russia

We study the spectral problems for integro-differential equations arising in the theory of Gaussian processes similar to the fractional Brownian motion. We generalize the method of Chigansky–Kleptsyna [1] and obtain two-term asymptotics for the eigenvalues of such equations. An application to the small ball probabilities in L_2-norm is described.

The talk is based on paper [2].

The research is supported by the joint RFBR–DFG grant 20-51-12004 NNIO.

References

[1] P. Chigansky and M. Kleptsyna. *Exact Asymptotics in Eigenproblems for Fractional Brownian Covariance Operators.* Stoch. Proc. Appl., 128(6):2007-2059, 2018.

[2] A. I. Nazarov. *Spectral Asymptotics for a Class of Integro-Differential Equations Arising in the Theory of Fractional Gaussian Processes.* arXiv: 1908.10299.

Luttinger–Sy model and ground state energy per particle in the thermodynamic limit

Vadim Ognov

IMJ-PRG, Sorbonne Université, France

In this talk, we will study a one-dimensional system of interacting electrons, the Luttinger–Sy model (or pieces model). Assuming a fast decay of the interactions, F. Klopp and N. A. Veniaminov proved the almost sure convergence of the thermodynamic limit of the ground state energy per particle. They also gave an expansion of this limit. We will assume that the interactions are compactly supported and present an improved expansion.

Analogs of Szegö's theorem for ergodic operators

Leonid Pastur

B. I. Verkin Institute for Low Temperatures Physics and Engineering, Kharkiv, Ukraine

We present a setting generalizing that for Szegö's theorem on the Töplitz (or discrete convolution) operators. Viewing the theorem as an asymptotic trace formula determined by a certain underlying operator and by two functions (the symbol and the test function), we replace the Töplitz operator by an ergodic operator (e.g. random or quasiperiodic), in particular, by the discrete Schrödinger operator with ergodic potential. In the framework of this setting, we discuss a variety of asymptotic formulas different from those given by Szegö's theorem and determined by the smoothness of the test function and the symbol as well as the spectral type of the corresponding underlying operator. The formulas include certain Central Limit Theorems in the spectral context and large block asymptotics for the entanglement entropy of free disordered fermions.

A modified method of the first approximation and operator-type estimates in the homogenization theory

Svetlana Pastukhova

Russian Technological University, Moscow

We discuss some ideas and results from the homogenization theory studying heterogeneous media (e.g. composites, porous media, perforated media, singular structures like graphs and others) depending on a small parameter ε, which characterizes the inhomogeneity (for instance, the heterogeneous media can be ε-periodic). The principal goal of the homogenization theory is to find a homogeneous medium that roughly can replace the heterogeneous one as ε tends to zero or, in other words, to find effective (or homogenized) characteristics of the heterogeneous medium that correspond to some homogeneous medium and are actually close to the caracteristics of the original one for sufficiently small ε.

Estimates for the homogenization errors have been the focus of attention from the very beginning, since the 60s when this theory emerged. But till the last decades, the error estimates were obtained under rather strong regularity conditions (on the coefficients of the PDEs, the initial data, the right-hand side functions). On contrary, as $\varepsilon \to 0$, results on the convergence obtained without estimating the convergence rate were proved in quite a general case.

Last two decades, *operator-type estimates of the homogenization errors* were proved under minimal regularity conditions on the data. Among the pioneer publications on the operator-type estimates for elliptic equations, we single out first of all [1] and [2], where L^2-estimates for the error terms of the order of ε were obtained, and approximations of the resolvent with respect to the operator L^2-norm up to terms of the order of ε were found.

There are two main approaches to obtain the operator-type estimates. One is an approach based on the Floquet–Bloch transformation and, therefore, tightly related to periodic problems. This approach is used by Birman and Suslina. There is another approach, called the modified method of the first approximation (MMFA). It uses completely different ideas and is proposed in [2]. Being rather simple conceptually, MMFA proved to be powerful, effective for various homogenization problems. One can use this method to treat a wide range of problems and to apply to problems without underlying periodicity. There appeared another version of MMFA [3] slightly different from the original one of [2].

In the talk we discuss operator-type estimates obtained by using MMFA; some of them are known, see e.g. review [4], and others are new [5].

References

[1] M. Sh. Birman and T. A. Suslina. *Second Order Periodic Differential Operators. Threshold Properties and Homogenization.* St. Petersburg Math. J. 15: 639–714, 2004.

[2] V. V. Zhikov. *On Operator Estimates in Homogenization Theory.* Dokl. Math. 72:535–538, 2005.

[3] V. V. Zhikov and S. E. Pastukhova. *On Operator Estimates for Some Problems in Homogenization Theory.* Russian Journal of Math. Physics, 12: 515–524, 2005.

[4] V. V. Zhikov and S. E. Pastukhova. *Operator Estimates in Homogenization Theory.* Russian Math. Surveys, 71: 417–511, 2016.

[5] S. E. Pastukhova. *L^2-estimates for Homogenization of Elliptic Operators.* Journal of Math. Sciences, 244:671–685, 2020.

Mathematical scattering theory for quantum waveguides

Alexander Poretskii

St. Petersburg State University, Russia

A waveguide is a domain with several cylindrical "ends". The wave propagation is described by a nonstationary equation of the form $i\partial_t f = \mathcal{A}f$, where $\mathcal{A}$ is a selfadjoint second order elliptic operator with variable coefficients (in particular, for $\mathcal{A} = -\Delta$, Δ being the Laplace operator, this equation is the Schrödinger equation). For the corresponding stationary problem with a spectral parameter, we define continuos spectrum eigenfunctions and a scattering matrix. The limiting absorption principle allows us to get the eigenfunction expansion. We also calculate wave operators and prove their completeness. Then we define a scattering operator and describe its connection with the scattering matrix. More details can be found in [1].

References

[1] B. A. Plamenevskii, A. S. Poretskii and O. V. Sarafanov. *Mathematical Scattering Theory in Quantum Waveguides.* Doklady Physics. 64:430–433, 2019.

On spectral asymptotics for the Sturm–Liouville problem with singular self-similar weight measure

Nikita Rastegaev

St. Petersburg State University, Russia

Eigenvalue asymptotics for the Sturm–Liouville problem

$$\begin{gathered} -y'' = \lambda \mu y, \\ y'(0) = y'(1) = 0, \end{gathered} \tag{1}$$

is well-studied in the case of a regular measure μ or a measure μ containing a regular component. It is more difficult to obtain good estimates when the measure μ is singular with respect to the Lebesgue measure. Results in this area go back to works of M. G. Krein, which show, in particular, that if μ contains a nonzero regular component, then its singular component does not affect the main term of the spectral asymptotics.

When considering a purely singular mesure, it is necessary to restrict the class of measures under consideration in order to gain nontrivial estimates. In particular, interesting results arise when considering measures with self-similar structure (e.g. measures the primitive of which is a Cantor type function). In [1] the power exponent of the main asymptotic term is obtained in the case of a self-similar measure. Later it was shown, see [2] and [3], for a singular self-similar measure, that the main term of the spectral asymptotics may contain not just the power component, but also a periodic function of a logarithm.

In [4] a class of measures was destribed for which this periodic component is non-constant. This talk is based on results of [5]–[7], we present a series of generalizations of the main result of [4] to gradually widening classes of measures. It will also concern a generalization of the result of [1] to the case of a measure self-similar with respect to a set of *nonlinear* contractions of $[0, 1]$.

The research was supported under President RF Grant 075-15-2019-204.

References

[1] T. Fujita. *A Fractional Dimension, Self-Similarity and a Generalized Diffusion Operator.* In: Taniguchi Symp. PMMP. Katata, 1985, 83–90.

[2] M. Solomyak, E. Verbitsky. *On a Spectral Problem Related to Self-Similar Measures.* Bull. London Math. Soc. 27:242–248, 1995.

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[7] U. R. Freiberg, N. V. Rastegaev. *On Spectral Asymptotics of the Sturm–Liouville Problem with Self-Conformal Singular Weight with Strong Bounded Distortion Property.* Zap. Nauchn. Sem. POMI, 477:129–135, 2018.

Spectral properties of the Neumann-Poincare operator for the elasticity system and related questions about zero order pseudodifferential operators

Grigori Rozenblum

St. Petersburg State University, Russia

The Neumann–Poincaré (NP) operator (called also the double layer potential) attracted recently attention due to its connection to properties of metamaterals. For the case of the Lamé system in 2D and 3D elasticity the NP operator turns out to be a zero order pseudodifferential operator. We discuss the location and some properties of the essential spectrum of this operator, which is quite different for homogeneous and nonhomogeneous bodies, as well as determine asymptotic properties of sequencies of eigenvalues converging to the points of essential spectrum. The rate of this convergence depends crucially on the structure of the essential spectrum.

The talk is based on a joint work with Yoshihisa Miyanishi.

Anderson localization beyond regular Floquet eigenvalues

Albrecht Seelmann

Technische Universität Dortmund, Germany

We prove that Anderson localization near band edges of multi-dimensional ergodic random Schrödinger operators with periodic background potential in $L^2(\mathbb{R}^d)$ is universal. By this we mean that Anderson localization holds without extra assumptions on the random variables and independently of regularity or degeneracy of the Floquet eigenvalues of the background operator. The main novelty is an initial scale estimate, the proof of which avoids Floquet theory altogether and uses instead an interplay between quantitative unique continuation and large deviation estimates.

This talk is based on joint work with Matthias Täufer (Queen Mary University, London).

On homogenization for locally periodic strongly elliptic operators

Nikita Senik

St. Petersburg State University, Russia

In the homogenization theory, one is interested in studying asymptotic properties of solutions to differential equations with rapidly oscillating coefficients. We will consider such a problem for a matrix strongly elliptic operator $\mathcal{A}^\varepsilon = -\operatorname{div} A(x, x/\varepsilon)\nabla$ on $\mathbb{R}^d$, where A is Hölder continuous of order $s \in [0, 1]$ in the first variable and periodic in the second. It is well known that the resolvent $(\mathcal{A}^\varepsilon - \mu)^{-1}$ converges, in some sense, as $\varepsilon \to 0$. In this talk, we will discuss results regarding convergence in the uniform operator topology on $L_2(\mathbb{R}^d)^n$, i.e., the strongest type of operator convergence. We present the first two terms of an approximation for $(\mathcal{A}^\varepsilon - \mu)^{-1}$, with particular attention paid to the rates of approximations.

On semiclassical methods for diference equations

Ekaterina Shchetka

St. Petersburg State University and St. Petersburg Department of Steklov Institute

We describe semiclassical asymptotics of analytic solutions to systems of first order difference equations in the complex plane. The asymptotics contain an analog of the Berry phase well known in the case of differential equations. The results were used to study the spectrum and density of states for the almost-Mathieu operator.

The talk is based on a joint work with Alexander Fedotov.

Constants in the Sobolev embedding theorems. Applications to spectral problems

Igor Sheipak

Lomonosov Moscow State University, Russia

We study the minimization problem, with respect to a parameter, for the eigenvalues of the spectral problem with weight generated by the kth derivative of the delta-interaction. It is shown that this problem is related to finding the sharp constant for the embedding of the Sobolev space $\mathring{W}_2^n[0;1]$ into $\mathring{W}_\infty^k[0;1]$, $0 \leqslant k < n$. This problem is in turn directly related to the question of describing properties of the functions that bound the kth derivatives of functions in the space $\mathring{W}_2^n[0;1]$, with $n > k$, by their $\mathring{W}_2^n$-norms.

We found explicit formulas for the extreme splines on which the upper bound is attained. We also obtained explicit representations of the estimation functions for $k = 3$ or $k = 5$, as well as even $k < n$. We have determined the properties of the extrema of these functions. In all these cases, the sharp embedding constants were found, and the smallest eigenvalue of the initial spectral problem was found.

On perturbations of self-adjoint and normal operators. Analytical aspects

Andrew Shkalikov

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We will talk about spectral properties of operators of the form $A = T + B$, where B is a non-symmetric operator subordinated to a self-adjoint or normal operator T. An operator B is said to be $T - p$-subordinated ($0 \leqslant p < 1$) if the domain of B contains the domain of T and

$$\|Bx\| \leqslant b\|Tx\|^p \, \|x\|^{1-p} \quad \forall\, x \in \mathcal{D}(T) \subset \mathcal{D}(B), \quad b = \text{const}.$$

There is a great number of results (in particular, by J. Birkhoff, T. Carleman, M. V. Keldysh, F. Brauder, S. Agmon, V. B. Lidskii, I. Ts. Gohberg and M. G. Krein, F. S. Markus, V. I. Matsaev, B. S. Mityagin) concerning completeness and basisness of the eigenfunctions and preserving of the eigenvalue asymptotics of self-adjoint operators under p-subordinated perturbations.

We introduce new concepts of local subordination and local subordination in the sense of quadratic forms and prove new theorems which involve new technique and can be applied to concrete problems in more general situation.

In the second part, we will discuss results on perturbations of a self-adjoint operator T with continuous spectrum whose spectrum consists of infinitely many segments $\{[\sigma_k,\ \sigma_{k+1}]\}_{k=1}^{\infty}$ separated by gaps: $\mathrm{dist}(\sigma_k, \sigma_{k+1}) \geqslant \mathrm{const}$.

The work is supported by RFBR (Russian Foundation for Basic Research, grant No 19-01-00240).

Preserving of the basis property under locally subordinated perturbations of self-adjoint operators

Vladimir Sivkin

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In this talk we study the basis properties of the root vectors of the operator $A = T + B$, where B is a non-self-adjoint operator locally p-subordinated to a self-adjoint operator T with a discrete spectrum. We find conditions which guarantee that root vectors of $T + B$ form a basis with parenthesis.

Estimates and asymptotic behavior of the discrete spectrum of a discrete periodic Schrödinger operator perturbed by a decreasing potential

Vladimir Sloushch

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Consider a discrete periodic Schrödinger operator $H := \Delta + Q$ on a locally finite connected $\mathbb{Z}^d$-periodic graph Γ embedded in $\mathbb{R}^d$; here Δ is the discrete Laplace operator on Γ, and Q is a real bounded $\mathbb{Z}^d$-periodic potential on Γ. The operator H is perturbed by a sign-definite decreasing potential V defined on the graph Γ. The potential V has a power-like asymptotics at infinity

$$0 \leq V(x) \sim \vartheta(\frac{x}{|x|})|x|^{-d/p}, \quad |x| \to \infty, \quad p > 0. \tag{2}$$

We are interested in the spectrum of operators $H_\pm(t) := H \pm tV$, $t > 0$, arising in spectral gaps of the operator H. Suppose the spectrum of the operator H contains the gap (Λ_+, Λ_-) (possibly semi-infinite). Since the potential V decreases on infinity, the spectrum of $H_\pm(t)$, $t > 0$, is discrete in the gap (Λ_+, Λ_-). Eigenvalues of the operators $H_\pm(t)$ move monotonously with increasing t. The main objects of our study are the counting functions $N_\pm(\lambda, \tau, V)$, $\lambda \in [\Lambda_+, \Lambda_-]$, $\tau > 0$, which are defined as the number of eigenvalues of the operators $H_\pm(t)$ passing through the point λ with increasing t from 0 to τ.

The main result of the work is as follows: *if the perturbation satisfies the condition* (2)*, then the counting functions have power-like asymptotics with respect to large coupling constant for all* $\lambda \in (\Lambda_+, \Lambda_-)$

$$N_\pm(\lambda, \tau, V) \sim \Gamma_p^\pm(\lambda, H, V)\tau^p, \quad \tau \to +\infty. \tag{3}$$

The coefficients $\Gamma_p^{\pm}(\lambda, H, V)$ can be calculated in terms of the operator H and the perturbation V. Under certain conditions the asymptotics (3) *holds at the edges of the gap.*

This work was done in collaboration with E. L. Korotyaev.

Asymptotic behavior of orthogonal polynomials without the Carleman condition

Dimitri Yafaev

St. Petersburg State University, Russia and Univ. of Rennes, France

Our goal is to find an asymptotic behavior as $n \to \infty$ of orthogonal polynomials $P_n(z)$ defined by the Jacobi recurrence coefficients a_n, b_n. We suppose that the off-diagonal coefficients a_n grow so rapidly that the series $\sum a_n^{-1}$ converges, that is, the Carleman condition is violated. With respect to diagonal coefficients b_n we assume that $-b_n(a_n a_{n-1})^{-1/2} \to 2\beta_\infty$ for some $\beta_\infty \neq \pm 1$. The asymptotic formulas obtained for $P_n(z)$ are quite different from the case $\sum a_n^{-1} = \infty$ when the Carleman condition is satisfied. In particular, if $\sum a_n^{-1} < \infty$, then the phase factors in these formulas do not depend on the spectral parameter $z \in \mathbb{C}$. The asymptotic formulas obtained in the cases $|\beta_\infty| < 1$ and $|\beta_\infty| > 1$ are also qualitatively different from each other. As an application of these results, we find necessary and sufficient conditions for the essential self-adjointness of the corresponding minimal Jacobi operator.

Anderson localization for random Dirac operators

Sylvain Zalczer

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The 2-dimensional Dirac operator is used to model the dynamics of an electron in a graphene sheet. Thus, it makes sense to model graphene with impurities by a randomly perturbed Dirac operator. We study Anderson localization, which is a usual property of disordered systems, in this setting. We prove it at the edge of the bands of the spectrum of a gapped Dirac Hamiltonian and show the Lipschitz continuity of the density of states.

This is a joint work with J.-M. Barbaroux and H. D. Cornean.

High-frequency diffraction by a non-smooth contour

Ekaterina Zlobina

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High-frequency diffraction of a plane wave by a contour with curvature having a singularity is addressed. In contrast with earlier research, the problem is considered in the framework of a rigorous boundary layer approach. An asymptotic description of wavefield within a boundary layer surrounding the point of non-smoothness of the contour is found and matched with wavefield near the limit ray.