Spectral theory and mathematical physics

Lecture mini-courses

September 2021

Frédéric Klopp. Recent results in localization.

The mini-course will be devoted to two recent results on localization.
The first result deals with one particle localization and is joint work with J. Schenker (https://arxiv.org/abs/2105.13215). It shows that, for quite general random models, while localization cannot be uniform on an non empty open interval of energy, a result which goes back to the 90's, it almost is i.e. only a small fraction of the states does not localize uniformly.
The second result deals with many body localization. For a simple one dimensional random Hamiltonian, we show that, in the thermodynamic limit:

- at zero temperature, for a small enough particle density, the ground state of the Hamiltonian of many fermions subjected to this one dimensional random Hamiltonian interacting through a compactly supported repulsive potential exhibits localization.
- at positive temperature, for a small enough chemical potential, the Gibbs state of the grand canonical ensemble for the same Hamiltonian exhibits localization.

Here, in both cases, a state "exhibits localization" if its two particle density matrix decays exponentially off the diagonal.

Vladimir Nazaikinskii. Geometry and semiclassical asymptotics

Maslov's canonical operator is one of the most powerful tools for constructing global semiclassical asymptotics for linear differential equations and systems. We will outline the rich geometry underlying the canonical operator (Lagrangian manifolds in the phase space, focal points, caustics, Maslov index, etc.) and explain its up-to-date construction suitable not only for theoretical research but also for the efficient analysis of specific problems using the capabilities of technical computation systems such as Wolfram Mathematica.

Alexander Pushnitski. Additive and multiplicative Hankel and Toeplitz operators

In the first part, I will discuss the classical (additive) Toeplitz and Hankel operators. These are operators whose matrix representations have the form {t(j-k)} for Toeplitz and {h(j+k)} for Hankel (here j,k are non-negative integers). In the second part, I will discuss the multiplicative Toeplitz and Hankel operators; these are operators represented by infinite matrices of the form {t(j/k)} and {h(jk)}, where j,k are natural numbers. It is well known that additive Toeplitz and Hankel operators can be naturally realised as operators on the Hardy space. It turns out that in a similar way the multiplicative Toeplitz and Hankel operators can be realised as operators acting on a certain Hilbert space built from Dirichlet series. I will discuss the general set-up for the theory of these classes of operators and mention some key questions: boundedness, compactness, finite rank property, etc.

	16:00-16:45 and 16:50-17:35	17:55-18:40 and 18:45-19:30
8, September	Vladimir Nazaikinskii	Alexander Pushnitski
15, September	Frederic Klopp	Alexander Pushnitski
29, September	Vladimir Nazaikinskii	Frederic Klopp