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Lieb-Thirring estimates for Schrödinger operators with singular measures as potential

L–T estimates for the Schrödinger operator $-\Delta - V$ in $\mathbb{R}^d$ concern the sum of powers of moduli of negative eigenvalues, $LT_\gamma(V) = \sum_j |\lambda_j|^\gamma$. The standard L-T estimate has the form

$$LT_\gamma(V) \leq \mathrm{L}_{\gamma,d} \int V_+(x)^{\frac{d}{2}+\gamma} dx,$$

which holds for $\gamma \geq 0$, $d > 2$; $\gamma > 0$, $d = 2$; $\gamma \geq \frac{1}{2}$, $d = 1$. These estimates play important role in the spectral theory of quantum, especially many-particle, systems and were being under active development during latest 50 years. In the talk we discuss some recent results on the L–T inequalities for Schrödinger-like operators with potential V replaced by a measure $V\mu$, where μ is a measure singular with respect to the Lebesgue measure and V is a μ-measurable function. Typical measures μ in this setting are the surface measure on a Lipschitz surface of dimension s in $\mathbb{R}^d$ or the Hausdorff measure on a fractal set. The general estimate for the operator $(-\Delta)^l - V(x)\mu$ has the form

$$\sum_j |\lambda_j|^\gamma \leq \mathrm{L}_{\gamma,d,l} \int V_+(x)^\theta \mu(dx),\ \theta = \frac{s + 2l\gamma}{s - d + 2l},$$

where s is the dimensional characteristic of the measure μ.